sample exam solutions

1

(a) $|A| = \frac{1}{2}|4-0| = 2$

$K = \underset{\text{max}}{4} - \underset{|A|}{2} = 2$

period $= \pi$

$\pi = \frac{2\pi}{B}$

$B = 2$

④ phase shift $= \frac{\pi}{4}$

$\frac{\pi}{4} = \frac{-C}{B}$

$\frac{\pi}{4} = \frac{-C}{2}$

$-\frac{\pi}{2} = C$

$$y = 2 - 2\sin\left(2x - \frac{\pi}{2}\right)$$

(b)

① $|A| = \frac{1}{2}|0-(-6)| = 3$

② $K = 0 - 3 = -3$

③ $A = 3$ not reflected

④ period $= \frac{2\pi}{3}$

$\frac{2\pi}{3} = \frac{2\pi}{B}$

$B = 3$

⑤ p.s. $= \frac{\pi}{6}$

$\frac{\pi}{6} = \frac{-C}{3}$

$\frac{-\pi}{2} = C$

$$y = -3 + 3\sin\left(3x - \frac{\pi}{2}\right)$$

(c)

① $|A| = \frac{1}{2}|0-(-4)| = 2$

② $K = 0 - 2 = -2$

③ $A = 2$ not reflected

④ period $= \frac{\pi}{2}$

$\frac{\pi}{2} = \frac{2\pi}{B}$

$B = 4$

⑤ phase shift $= -\frac{\pi}{4}$

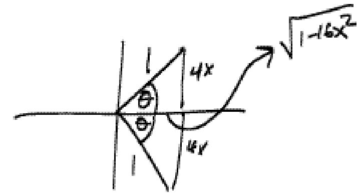
$-\frac{\pi}{4} = \frac{-C}{4}$

$\pi = C$

$$y = -2 + 4\cos(4x + \pi)$$

3 – 7, see book or handouts

(8) $\tan\left(\sin^{-1} 4x\right)$

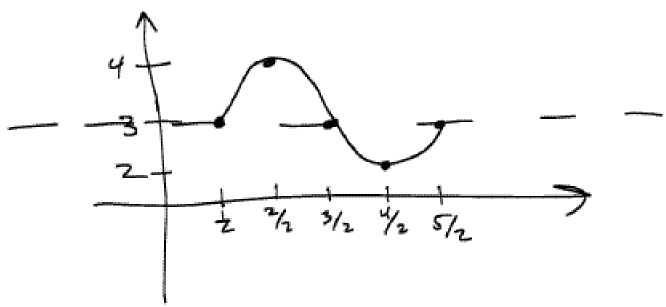


$\theta = \sin^{-1} 4x \longrightarrow \sin\theta = \frac{4x}{1}$

$$\boxed{\tan\theta = \frac{4x}{\sqrt{1-16x^2}}}$$

Calculator Section

(1) $y = 3 + \sin\left(\pi x - \frac{\pi}{2}\right)$



(1) $VT = 3$

(2) $|A| = 1$

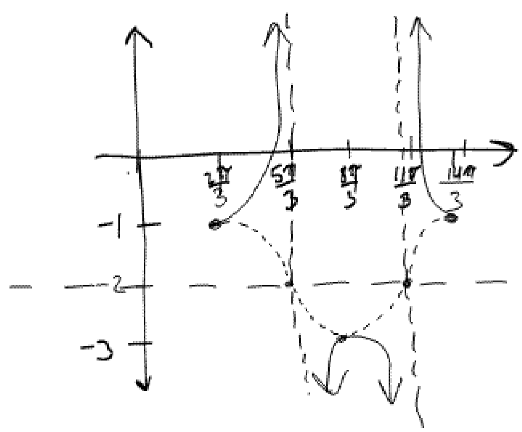
(3) period $= \frac{2\pi}{\pi} = 2$, $QP = \frac{1}{2}$

(4) one cycle: $0 \leq \pi x - \frac{\pi}{2} \leq 2\pi$

$\frac{\pi}{2} \leq \pi x \leq \frac{5\pi}{2}$

$\frac{1}{2} \leq x \leq 5/2$

(2) $y = -2 + \sec\left(\frac{1}{2}x - \frac{\pi}{3}\right)$



(1) $VT = -2$

$|A| = 1$

period $= \frac{2\pi}{1/2} = 4\pi$, $QP = \pi = \frac{3\pi}{3}$

one cycle:

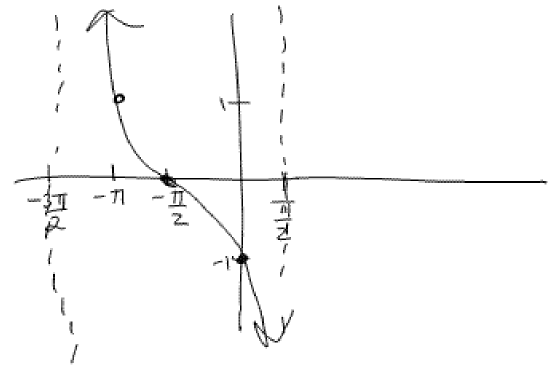
$0 \leq \frac{1}{2}x - \frac{\pi}{3} \leq 2\pi$

$\frac{\pi}{3} \leq \frac{1}{2}x \leq \frac{7\pi}{3}$

$\frac{2\pi}{3} \leq x \leq \frac{14\pi}{3}$

Hw 7 (5.4 - 6.2) 20 20 2/2/2010 Homework Can be dropped Raw Scores Include Assignment

③ $y = -1 - \tan\left(\frac{1}{2}x + \frac{\pi}{4}\right)$



① vt = -1

(2) $|A| = 1$, reflected

③ period $= \frac{\pi}{\frac{1}{2}} = 2\pi$, $QP = \frac{\pi}{2}$

one cycle:

$-\frac{\pi}{2} < \frac{1}{2}x + \frac{\pi}{4} < \frac{\pi}{2}$

$-\frac{3\pi}{4} < \frac{1}{2}x < \frac{\pi}{4}$

$-\frac{3\pi}{2} < x < \frac{\pi}{2}$

④

ⓐ $\sin 75^\circ = \sin\left(\frac{150^\circ}{2}\right) = \sqrt{\frac{1 - \cos 150^\circ}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

or $= \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

ⓑ $\tan 105^\circ = \tan\left(\frac{210^\circ}{2}\right) = \frac{1 - \cos 210^\circ}{\sin 210^\circ} = \frac{1 + \frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\frac{2+\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = -2+\sqrt{3}$

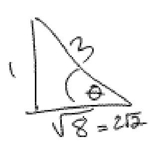
or $= \tan(60^\circ + 45^\circ) = \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60 \tan 45^\circ} = \frac{\sqrt{3}+1}{1-\sqrt{3}}$

⑥

ⓐ $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}} = \sqrt{\frac{1 + \frac{2\sqrt{2}}{3}}{2}}$

$= \sqrt{\frac{3+2\sqrt{2}}{6}}$

$\frac{\theta}{2} \in Q\text{II}$



⑥ ⓑ

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} = \frac{\frac{3-2\sqrt{3}}{3}}{-\frac{1}{3}} = -3+2\sqrt{3}$$

⑦ $\cos 2x \sin 8x = \frac{1}{2}\left[\sin(2x+8x) - \sin(2x-8x)\right]$

$= \frac{1}{2}\sin 10x + \frac{1}{2}\sin 6x$, sine is odd

⑧ $\sin 75^\circ - \sin 15^\circ = 2\cos\left(\frac{75^\circ+15^\circ}{2}\right)\sin\left(\frac{75^\circ-15^\circ}{2}\right)$

$= 2\cos\left(\frac{90^\circ}{2}\right)\sin\left(\frac{60^\circ}{2}\right)$

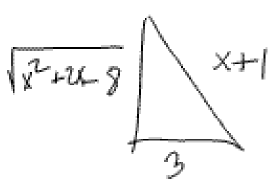
$= \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\cdot\frac{1}{2}$

$= \frac{\sqrt{2}}{2}$

⑨

$\sin\left(\sec^{-1}\frac{x+1}{3}\right) = \sin\theta = \frac{\sqrt{x^2+2x-8}}{x+1}$

$\theta = \sec^{-1}\frac{x+1}{3} \rightarrow \sec\theta = \frac{x+1}{3} \rightarrow \cos\theta = \frac{3}{x+1}$



$\sqrt{(x+1)^2-9} = \sqrt{x^2+2x+1-9}$

$= \sqrt{x^2+2x-8}$

(10) $\dfrac{\cos t}{1+\sin t} = \dfrac{1-\sin t}{\cos t}$

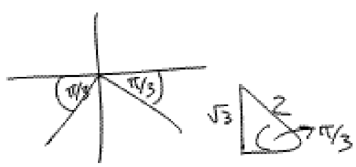
LHS

$$\frac{\cos t}{1+\sin t} = \frac{\cos t\,(1-\sin t)}{(1+\sin t)(1-\sin t)} = \frac{\cos t\,(1-\sin t)}{1-\sin^2 t} = \frac{\cancel{\cos t}(1-\sin t)}{\cos^2 t}$$

$$= \frac{1-\sin t}{\cos t} \checkmark$$

(11)

(a) $\sqrt{3} + 5\sin t = 3\sin t$



$2\sin t = -\sqrt{3}$

$\sin t = -\dfrac{\sqrt{3}}{2}$

$\boxed{t = \dfrac{4\pi}{3} \quad \text{or} \quad t = \dfrac{5\pi}{3}}$ $\quad 0 \le t < 2\pi$

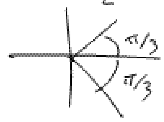
$\boxed{t = \dfrac{4\pi}{3} + 2\pi k \quad \text{or} \quad t = \dfrac{5\pi}{3} + 2\pi k}$ all radian solutions

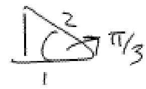
(b) $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ or $\cos x = -1$

$\cos x = \frac{1}{2}$





$\boxed{x = \pi/3 \quad \text{or} \quad x = \frac{5\pi}{3}}$

$\boxed{x = \frac{\pi}{3} + 2\pi k \quad \text{or} \quad x = \frac{5\pi}{3} + 2\pi k}$

(12)

(a) $\cos 2x - \cos x - 2 = 0$

$2\cos^2 x - \cos x - 3 = 0$

$(2\cos x - 3)(\cos x + 1) = 0$

$\cancel{\cos x = \frac{3}{2}}$ or $\cos x = -1$

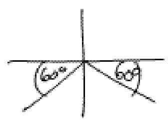
$3/2$ not $\in [-1,1]$ $\boxed{x = 180°}$

(b) see notes from Section 6.2

(13) $\sin 2x = -\frac{\sqrt{3}}{2}$

$u = 2x$

$\sin u = -\frac{\sqrt{3}}{2}$



$u = 240° + 360°k$ or $u = 300° + 360°k$

$2x = 240° + 360°k$ or $2x = 300° + 360°k$

$\boxed{x = 120° + 180°k \quad \text{or} \quad x = 150° + 180°k}$ k any integer

(14)

(a) $x = 2\cos t$ $y = 2\sin t$

$\frac{x}{2} = \cos t$ $\frac{y}{2} = \sin t$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\frac{x^2}{4} + \frac{y^2}{4} = 1$

$\boxed{x^2 + y^2 = 4}$ circle

(b) $x = 3\sin t$ $y = 4\cos t$

$\frac{x}{3} = \sin t$ $\frac{y}{4} = \cos t$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

$\frac{x^2}{9} + \frac{y^2}{16} = 1$ ellipse

(c) $x = \cos t - 3$ $\quad$ $y = \sin t + 2$

$x + 3 = \cos t$ $\quad$ $y - 2 = \sin t$

$(x+3)^2 + (y-2)^2 = 1$ circle

(d) $x = 3\cot t$ $\quad$ $y = 3\csc t$ $\quad$ $\cot^2 t + 1 = \csc^2 t$

$\frac{x}{3} = \cot t$ $\quad$ $\frac{y}{3} = \csc t$ $\quad$ $\csc^2 t - \cot^2 t = 1$

$$\left(\frac{y}{3}\right)^2 - \left(\frac{x}{3}\right)^2 = 1$$

$\frac{y^2}{9} - \frac{x^2}{9} = 1$ hyperbola