Math 150 Exam 2 Winter 2010

Name:_____

You must show all work to receive credit for correct and incorrect work. Good luck.

Calculator Section

(1) Find the exact value by using a **half angle** formula.

$\tan(15°) = \frac{1-\cos 30^\circ}{\sin 30^\circ} = \frac{1-\frac{\sqrt{3}}{2}}{1/2} = 2-\sqrt{3}$

(2) Find the exact value using a **sum** formula.

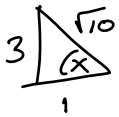
$\sin(105°) = \sin(60^\circ+45^\circ) = \sin 60^\circ\cos 45^\circ + \sin 45^\circ \cos 60^\circ$

$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

(3) Let $\cos x = \frac{1}{\sqrt{10}}$ with x in QIV, find

$\tan 2x = \frac{2\tan A}{1-\tan^2 A} = \frac{2(-3)}{1-(-3)^2} = \frac{-6}{1-9} = \frac{-6}{-8} = \frac{3}{4}$

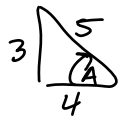


$\tan x = -3$

(4) If $\sin A = \frac{-3}{5}$ with A in QIII, find

$\frac{A}{2} \in QII$ $\quad \cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} = -\sqrt{\frac{1-\frac{4}{5}}{2}}$

$= -\frac{1}{\sqrt{10}}$



$\cos A = -\frac{4}{5}$

(5) Rewrite each expression as a sum or difference, then simplify.

$\sin 40^\circ \sin 35^\circ = \frac{1}{2}\left[\cos(40^\circ - 35^\circ) + \cos(40^\circ + 35^\circ)\right]$

$= \frac{1}{2}\left[\cos 5^\circ + \cos 75^\circ\right]$

(6) Rewrite as a product. Simplify if possible.

$\cos\frac{\pi}{12} + \cos\frac{7\pi}{12} = 2\cos\left(\frac{\frac{\pi}{12}+\frac{7\pi}{12}}{2}\right)\cos\left(\frac{\frac{\pi}{12}-\frac{7\pi}{12}}{2}\right)$

$= 2\cos\left(\frac{2\pi}{6}\right)\cos\left(-\frac{\pi}{4}\right)$

$= 2\cos\left(\frac{\pi}{3}\right)\cos(\pi/4)$

$= 2\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2}$

(7) Verify the identity.

$$\frac{\sec\theta + 1}{\tan\theta} = \frac{\tan\theta}{\sec\theta - 1}$$

LHS

$\frac{\sec\theta+1}{\tan\theta} = \frac{\sec\theta+1}{\tan\theta}\cdot\frac{\sec\theta-1}{\sec\theta-1}$

$= \frac{\sec^2\theta - 1}{\tan\theta(\sec\theta - 1)}$

$= \frac{\tan^2\theta}{\tan\theta(\sec\theta - 1)}$

$= \frac{\tan\theta}{\sec\theta - 1}$ ✓

(8) Find all radian solutions.

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

$x = \frac{7\pi}{6} + 2\pi k$ or $x = \frac{\pi}{2} + 2\pi k$, k any integer

or

$x = \frac{11\pi}{6} + 2\pi k$

(9) Solve for x if, $0° \le x < 360°$.

$4\sin^2 x + 4\cos x - 5 = 0$

$4(1 - \cos^2 x) + 4\cos x - 5 = 0$

$4 - 4\cos^2 x + 4\cos x - 5 = 0$

$-4\cos^2 x + 4\cos x - 1 = 0$

$4\cos^2 x - 4\cos x + 1 = 0$

$(2\cos x - 1)(2\cos x - 1) = 0$

$\cos x = \frac{1}{2}$

$x = 60°$

or

$x = 300°$

(10) Solve for x if, $0 \le x < 2\pi$

$\cos 3x = \frac{1}{2}$

$3x = \frac{\pi}{3} + 2\pi k \longrightarrow x = \frac{\pi}{9} + \frac{2\pi k}{3}$

or

$3x = \frac{5\pi}{3} + 2\pi k \quad$ or $\quad x = \frac{5\pi}{9} + \frac{2\pi k}{3}$

$k = 0$	$k = 1$	$k = 2$
$x = \frac{\pi}{9}$	$x = \frac{7\pi}{9}$	$x = \frac{13\pi}{9}$
or	or	or
$x = \frac{5\pi}{9}$	$x = \frac{11\pi}{9}$	$x = \frac{17\pi}{9}$

(11) Eliminate the parameter t.

$x = \sin t - 2$

$y = \cos t - 3$

$\sin t = x + 2$

$\cos t = y + 3$

$\longrightarrow (x+2)^2 + (y+3)^2 = 1$

(12) Eliminate the parameter t.

$x = 3\sec t$

$y = 3\tan t$

$\sec t = \frac{x}{3}$

$\tan t = \frac{y}{3}$

$\left(\frac{x}{3}\right)^2 - \left(\frac{y}{3}\right)^2 = 1$

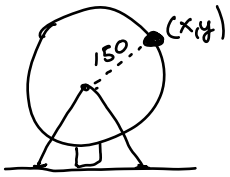
$\frac{x^2}{9} - \frac{y^2}{9} = 1$

Extra Credit:

Modeling a Ferris wheel

Suppose that a Ferris wheel has a radius of 150 feet and the distance from bottom of the Ferris wheel to the ground is 14 feet. Derive the parametric equations that will model a riders position on the Ferris wheel for any given angle *t*.

Note: For max credit the equations should model the Ferris wheel "realistically".



2pts

model the circle with radius 150 w/ parametric equations

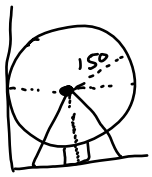
From Def 1

$\cos t = \frac{x}{r} \rightarrow x = r\cos t \rightarrow x = 150\cos t$

$\sin t = \frac{y}{r} \quad y = r\sin t \rightarrow y = 150\sin t$

4pts

model horizontal and vertical distance



Center: $(150, 164)$

radius 150

$(x-150)^2 + (y-164)^2 = 150^2$

$\left(\frac{x-150}{150}\right)^2 + \left(\frac{y-164}{150}\right)^2 = 1$

$\cos t = \frac{x-150}{150} \quad \sin t = \frac{y-164}{150}$

$x = 150\cos t + 150$

$y = 150\sin t + 164$

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No Calculator Section

(1) Give the following Sum or Difference formulas.

$\cos(A+B) =$ $\cos A\cos B - \sin A\sin B$

$\cos(A-B) =$ $\cos A\cos B + \sin A\sin B$

$\sin(A+B) =$ $\sin A\cos B + \sin B\cos A$

$\sin(A-B) =$ $\sin A\cos B - \sin B\cos A$

$\tan(A+B) =$ $\dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B)$ $\dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

(2) Give the following Double angle formulas.

$\sin 2A =$ $2\sin A\cos A$

$\cos 2A =$ $\cos^2 A - \sin^2 A$

$=$ $2\cos^2 A - 1$

$=$ $1 - 2\sin^2 A$

$\tan 2A =$ $\dfrac{2\tan A}{1 - \tan^2 A}$

(3) Give the following Half angle formulas.

$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$ $\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$

$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$ $\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$

(4) Evaluate without a calculator.

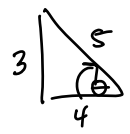
(a) $\cos\left(\cos^{-1}\frac{7\pi}{4}\right)$ = undefined

(b) $\cot\left(\tan^{-1}\frac{1}{2}\right) = \frac{1}{\tan(\tan^{-1}\frac{1}{2})} = \frac{1}{\frac{1}{2}} = 2$

Domain $\cos^{-1}$ is $[-1,1]$

$\frac{7\pi}{4} > 1$

(c) $\tan\left(\sin^{-1}\frac{3}{5}\right) = \frac{3}{4}$



$\theta = \sin^{-1}\frac{3}{5}$

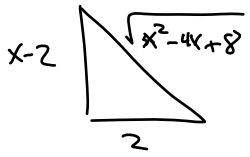
(d) $\sin^{-1}(\sin 225°) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$= -\frac{\pi}{4}$

(5) Write the expression as an equivalent expression involving x only. (Assume all variables represent a positive number.)

$$\sec\left(\tan^{-1}\frac{x-2}{2}\right) = \frac{\sqrt{x^2-4x+8}}{2}$$

$\theta = \tan^{-1}\frac{x-2}{2}$



(6) Graph one complete cycle of the equation below. You **must** label all five key points.

$$y = 3 + 2\cos\left(\frac{1}{2}x - \frac{\pi}{2}\right)$$

VT = 3

|A| = 2

per = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

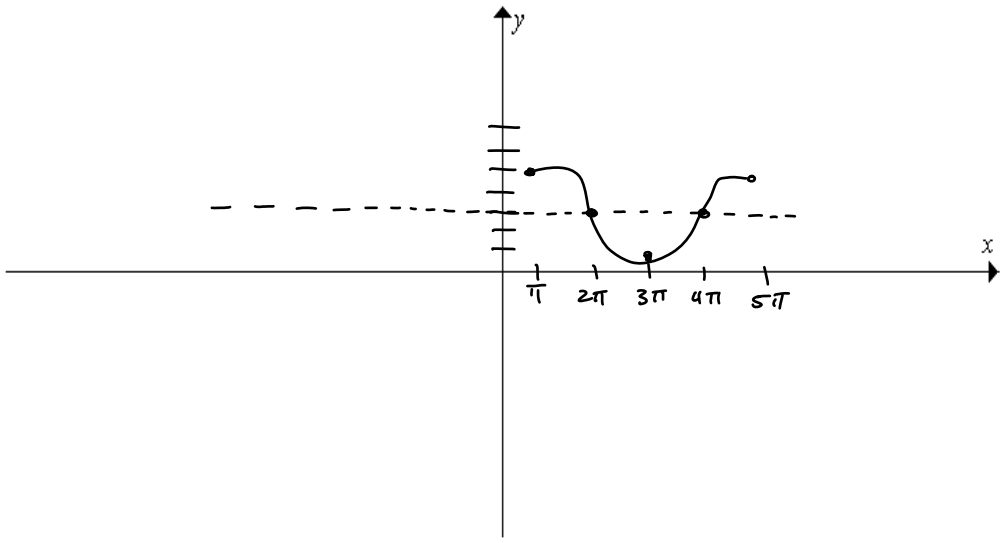
QP = π

one cycle

$0 \le \frac{1}{2}x - \frac{\pi}{2} \le 2\pi$

$\frac{\pi}{2} \le \frac{1}{2}x \le \frac{5\pi}{2}$

$\pi \le x \le 5\pi$



(7) Graph one complete cycle of the equation below. You **must** label all five key points.

$$y = -2 + \csc\left(\frac{1}{2}x - \frac{\pi}{3}\right)$$

VT = −2

|A| = 1

per $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

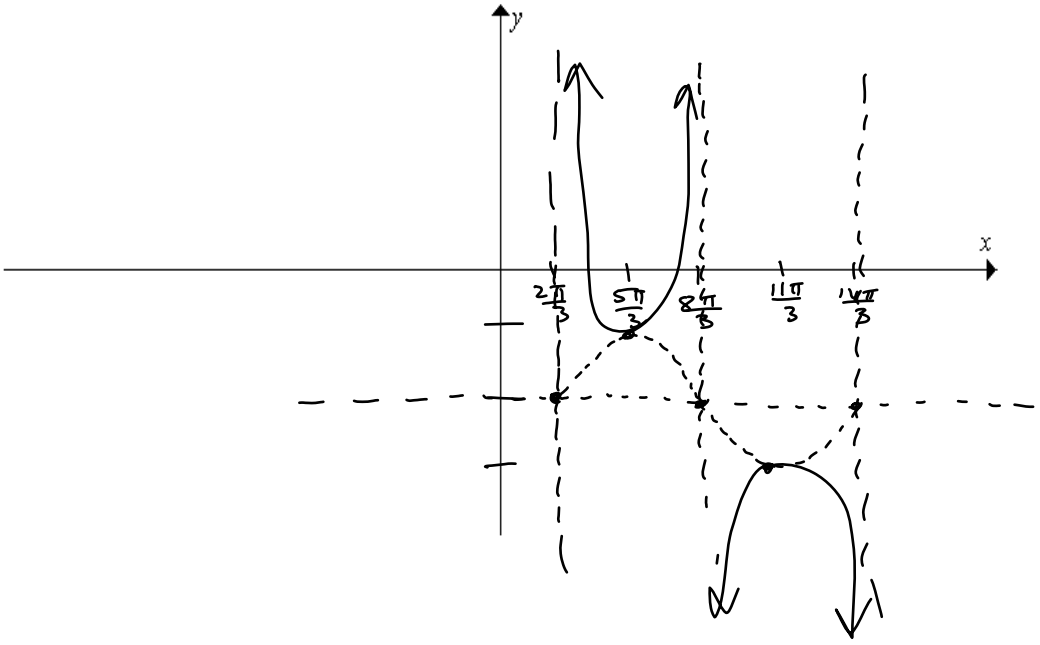
QP $= \pi = \frac{3\pi}{3}$

one cycle

$0 \le \frac{1}{2}x - \frac{\pi}{3} \le 2\pi$

$\frac{\pi}{3} \le \frac{1}{2}x \le \frac{7\pi}{3}$

$\frac{2\pi}{3} \le x \le \frac{14\pi}{3}$



(8) Graph one complete cycle of the equation below. You **must** label all five key points.

$$y = 1 + \tan\left(2x - \frac{\pi}{4}\right)$$

VT = 1

|A| = 1

Per $= \frac{\pi}{2}$

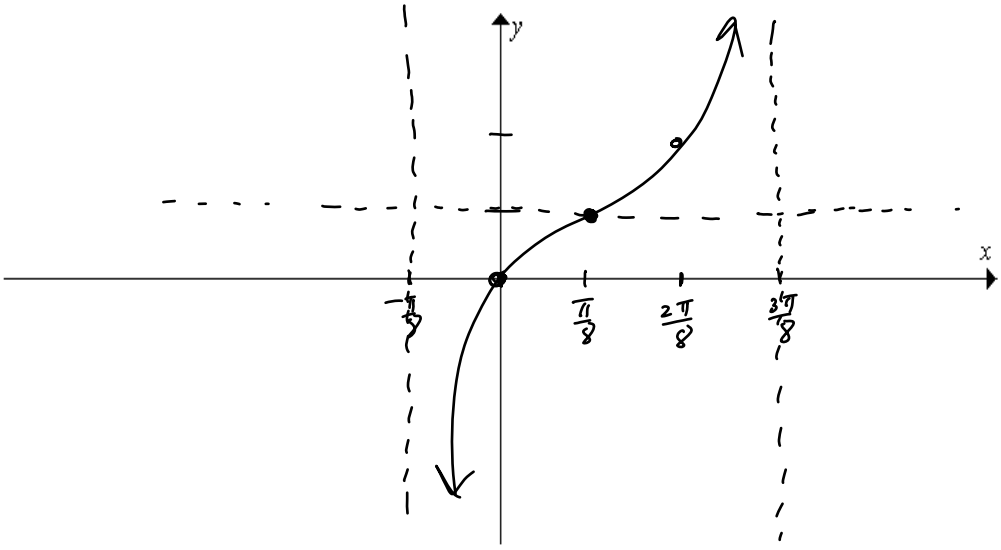
QP $= \frac{\pi}{8}$

One cycle

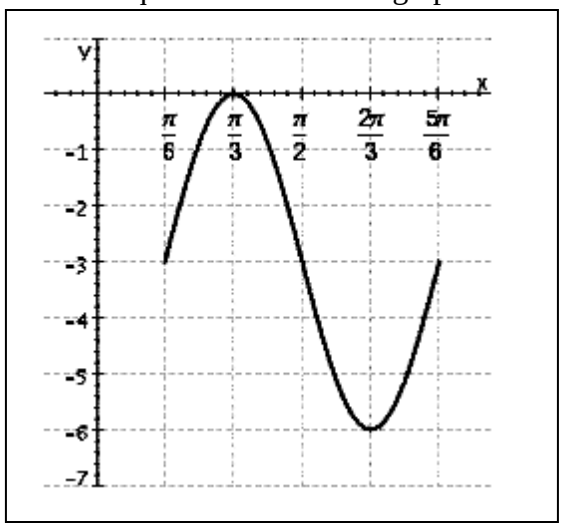
$-\frac{\pi}{2} \le 2x - \frac{\pi}{4} \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le 2x \le \frac{3\pi}{4}$

$-\frac{\pi}{8} \le x \le \frac{3\pi}{8}$



5 (9) The graph below is one complete cycle of the graph of an equation containing a trigonometric function. Find an equation o match the graph.



$|A| = \frac{1}{2}\left|0-(-6)\right| = 3$

$K = 0 - 3 = -3$

period $= \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

$\frac{2\pi}{3} = \frac{2\pi}{B}$

$B = 3$

phase shift $= \frac{\pi}{6}$

$\frac{\pi}{6} = \frac{-C}{3}$

$-\frac{3\pi}{6} = C$

$-\frac{\pi}{2} = C$

$y = -3 + 3\sin\left(3x - \frac{\pi}{2}\right)$